

I STARTED BY DRAWING TWO DIAGRAMS OF THE GIVEN BOTTLE. DIAGRAM (1) REPRESENTS ONLY THE INNER LINING OF THE BOTTLE WHICH SHOULD BE USED TO FIND THE VOLUME OF THE LIQUID. THIS WILL HELP TO MORE ACCURATELY DESCRIBE THE VOLUME OF THE LIQUID RATHER THAN MEASURING THE DIMENSIONS ON THE OUTSIDE SURFACE OF THE BOTTLE OF SODAS.

THE TOP OF DIAGRAM (1) ONLY GOES UP TO WHERE THE LIQUID STOPS SO AS TO NOT MEASURE THE EMPTY SPACE IN THE BOTTLE. DIAGRAM (2) USED TO CHECK SOME OF THE MEASUREMENTS.

I THEN DECIDED TO SPLIT THE LIQUID IN THE BOTTLE INTO FOUR SEPARATE POLYGONAL SHAPES. BY SEPARATELY FINDING THE VOLUME OF EACH OF THESE SPACE FIGURES, I WILL ADD ^{THE} THESE TO GET THE TOTAL VOLUME OF LIQUID IN THE SODA BOTTLE.

IN DIAGRAM (1), I NOTICED THAT AT THE TOP OF THE BOTTLE I COULD PARTITION THIS PORTION OF THE LIQUID INTO A CYLINDER REPRESENTED BY A. (IT CAN BE NOTED THAT I WILL USE INCHES AS A UNITS OF MEASURE. A RULER WAS USED FOR MEASURING). USING A RULER, I MEASURED THE DIAMETER OF THIS CYLINDER TO BE

$$d = \frac{11}{16} \text{ in}$$

TAKING HALF OF THIS, I GOT THE RADIUS TO BE:

$$r = \frac{11}{32} \text{ in}$$

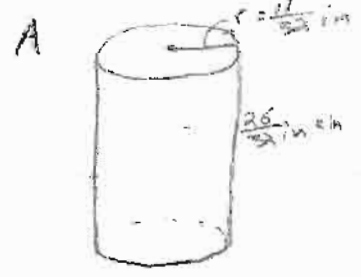
I ALSO MEASURED THE HEIGHT TO BE:

$$h = \frac{36}{32} \text{ in}$$

(SEE FIGURE A AT LEFT)

NOW USING THE FORMULA FOR THE VOLUME OF A CYLINDER:

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \left(\frac{11}{32} \text{ in}\right)^2 \left(\frac{36}{32} \text{ in}\right) \\ &\approx .418 \text{ in}^3 \end{aligned}$$



SO THE VOLUME OF THIS CYLINDER IS: $\approx .418 \text{ in}^3$

I THEN NOTICED THAT B IS A HALF OF A SPHERE.
I MEASURED THE RADIUS OF THIS SPHERE TO BE
 $r = \frac{36}{32}$ in. (SEE FIGURE B AT THE LEFT)



SINCE THE VOLUME OF A SPHERE IS

$$V = \frac{4}{3} \pi r^3$$

THEN HALF OF A SPHERE HAS VOLUME

$$V = \frac{1}{2} \left(\frac{4}{3} \right) \pi r^3$$

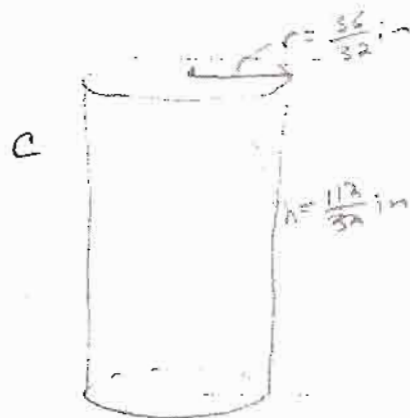
$$V = \frac{2}{3} \pi r^3$$

$$= \frac{2}{3} \pi \left(\frac{36}{32} \right)^3$$

SO THE VOLUME OF THIS
1/2 SPHERE IS:

$$B = 2.982 \text{ in}^3$$

THE PORTION OF THE BOTTLE LABELED C WAS ALSO
A CYLINDER (SEE FIGURE C AT LEFT). SIMILARLY
TO PART A, I MEASURED THIS TO BE:



$$V = \pi r^2 h$$

$$= \pi \left(\frac{36}{32} \text{ in} \right)^2 \left(\frac{112}{32} \text{ in} \right)$$

SO THE VOLUME OF THIS CYLINDER IS

$$C = 13.916 \text{ in}^3$$

THE BOTTOM OF THE BOTTLE IS SHAPED LIKE THE FRUSTUM
OF A CONE. AS SEEN IN THE DIAGRAM ON THE LEFT
I FOUND THE MEASUREMENTS TO BE

$$\text{RADIUS OF TOP} = \frac{36}{32} \text{ in}$$

$$\text{RADIUS OF BOTTOM} = \frac{29}{32} \text{ in}$$

$$\text{HEIGHT} = \frac{12}{32} \text{ in}$$

USING EACH RADIUS, I FOUND THE MEAS OF THE CIRCLES
OF THE TOP AND BOTTOM OF THE CONE TO BE:

$$\text{AREA OF TOP} = \pi \left(\frac{36}{32} \text{ in} \right)^2$$

$$A \approx 3.976 \text{ in}^2$$

$$\text{AREA OF BOTTOM} = \pi \left(\frac{29}{32} \text{ in} \right)^2$$

$$B \approx 2.50 \text{ in}^2$$

$$\sqrt{AB} = 3.203 \text{ in}$$

SO BY THE VOLUME FORMULA FOR THE FRUSTUM OF A CONE

$$V = \frac{1}{3} h (A + B + \sqrt{AB})$$

$$= \frac{1}{3} \left(\frac{12}{32} \text{ in} \right) (3.976 \text{ in}^2 + 2.50 \text{ in}^2 + 3.203 \text{ in}^2)$$

$$= 1.229 \text{ in}^3$$

SO THE VOLUME OF THE FRUSTUM OF THE CONE IS

$$D \approx 1.230 \text{ in}^3$$

By adding these four volumes together, I found the total volume of the fluid in the bottle to be

$$\begin{aligned}V &= A + B + C + D \\ &= 0.418 \text{ in}^3 + 2.982 \text{ in}^3 + 13.916 \text{ in}^3 + 1.220 \text{ in}^3 \\ &= 18.536 \text{ in}^3\end{aligned}$$

I felt there were four ways in which the estimate would yield less than perfect results. One was in any human error of measuring with the ruler. I tended to measure to the $\frac{1}{8}$ of an inch then convert smallest (when needed). Two, the volume of the half sphere was slightly over estimated since the top of the sphere violates the small cylinder up top (readily measured). Three, there is a slight curvature in the bottle which forced me to estimate the bottom using a frustum of a cone. This also adds a slight overestimation. Lastly, after converting fractions to decimals, I rounded numbers to the nearest thousandth.