Section IV Evidence for Meeting Standards
#6 Additional Assessment that Addresses NCTM Standards

1. Description of Department Algebra & Trigonometry Examination

This assessment is a department exam consisting of 20 questions drawn from high school algebra and trigonometry, 10 per area, that all candidates should know and be able to do, but may have forgotten during their college years. Task areas from algebra are completed without the aid of a calculator and include: solve absolute value inequalities; solve formulas for variables; solve systems of equations; use equations to find characteristics of graphs [e.g., intercepts, centers, minimums]; find functions whose graphs contain given points; solve exponential and logarithmic equations; and solve word problems. Tasks from trigonometry that are completed without the aid of a calculator include: convert between radians and degrees; find values of trig functions given angles and/or values of other trig functions; simplify trig expressions; verify trig identities; solve trig equations; convert between trig functions and their graphs. The following trigonometry tasks are completed with a calculator: find measures of angles and sides of given triangles; find values of angles for given values of trig functions.

2. Alignment of Department Algebra & Trigonometry Examination with NCTM Standards and Indicators

Program Standard	Indicators Addressed
Standard 1: Knowledge of Mathematical Problem Solving	1.1
Standard 3: Knowledge of Mathematical Communication	3.1, 3.2, 3.3
Standard 5: Knowledge of Mathematical Representation	5.2, 5.3
Standard 6: Knowledge of Technology	6.1
Standard 9: Knowledge of Number and Operation	9.1, 9.2, 9.4
Standard 10: Knowledge of Different Perspectives on Algebra	10.1, 10.5
Standard 15: Knowledge of Measurement	15.2

3. Data Results

Every candidate must pass this examination prior to entering SED 410 Practicum. Candidates have two opportunities to earn an overall score of 70 with a minimum score of 35 in each topic area. Candidates who are not successful after two attempts work with their advisor to determine an appropriate remediation plan and, upon completion, are offered one additional opportunity to pass the exam. Candidates who are unsuccessful on this third attempt are counseled out of the program.

Thus, 100% of the candidates who complete the program have successfully passed this exam before they enter Practicum.

4. Data Interpretation

Mathematics education faculty are committed to candidates' knowing the mathematics content they may be asked to teach during Practicum and student teaching. They established the examination in 2002 in response to errors that they observed candidates make in the field. The passing scores were set after several years of data and were adjusted to current levels in 2007. A sample exam is available to candidates so they can prepare; however, some candidates do not take the assessment seriously at first and are dismayed when they are not successful. In the past three years, in spite of remediation, three candidates were unable to achieve passing scores and were counseled out of the program.

Overall, faculty report that the exam is useful to ensure that candidates are prepared in the content they will teach in Practicum and student teaching and to enable candidates who are underprepared to remediate deficiencies that exist prior to entering Practicum.

5. Assessment Documentation

   a. Algebra-Trigonometry Exam

MATHEMATICS & COMPUTER SCIENCE DEPARTMENT

ALGEBRA &TRIGONOMETRY EXAMINATION

SAMPLE WITH SOLUTIONS

Algebra Non-calculator exam.

1. Solve for x: $\left|-\frac{5}{8}x+\frac{7}{2}\right|>1$.

$$-\frac{5}{8}x+\frac{7}{2}>1 \quad \text{or} \quad -\frac{5}{8}x+\frac{7}{2}<-1$$

$$-\frac{5}{8}x>-\frac{5}{2} \quad \text{or} \quad -\frac{5}{8}x<-\frac{9}{2}$$

$$x<4 \quad \text{or} \quad x>\frac{36}{5}$$

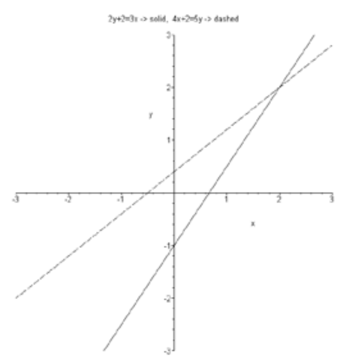
2. Solve for d: $p=\frac{2d(4-p^2)}{p+d}$

$$p = \frac{2d(4-p^2)}{p+d}$$
$$p^2 + pd = 8d - 2p^2 d$$
$$(2p^2 + p - 8)d = -p^2$$
$$d = \frac{-p^2}{2p^2 + p - 8}$$
$$d = \frac{p^2}{8 - p - 2p^2}$$

3. Solve the following system of equations algebraically and sketch a graph of the system:

$$\mathbf{2y + 2 = 3x}$$
$$\mathbf{4x + 2 = 5y}.$$

$$2y + 2 = 3x \Rightarrow -3x + 2y = -2 \Rightarrow -12x + 8y = -8$$
$$4x + 2 = 5y \Rightarrow 4x - 5y = -2 \Rightarrow \underline{12x - 15y = -6}$$
$$-7y = -14$$
Therefore, $y = 2 \Rightarrow 4x + 2 = 5(2) \Rightarrow x = 2,\ (x,y)=(2,2)$.



4. Find the center and radius of the circle with equation:

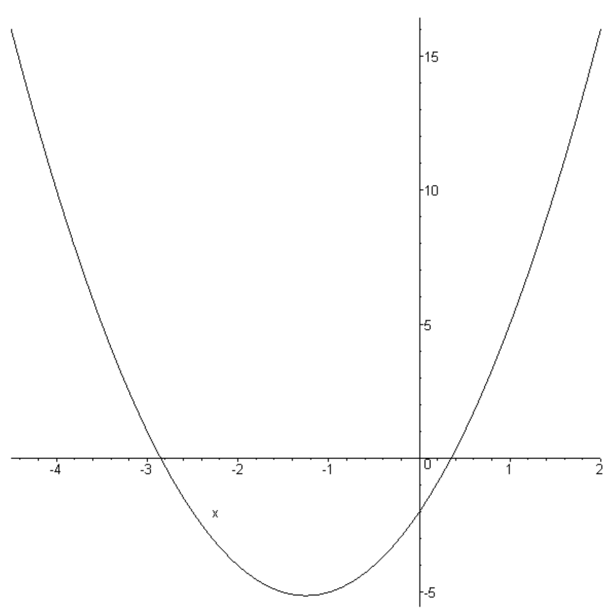
$$\mathbf{4x^2 + 4y^2 + 16x - 24y = 69.}$$

$$4x^2 + 4y^2 + 16x - 24y = 69$$
$$4(x^2 + 4x) + 4(y^2 - 6y) = 69$$
$$4(x^2 + 4x + 4) + 4(y^2 - 6y + 9) = 69 + 52$$
$$4(x + 2)^2 + 4(y - 3)^2 = 121$$
$$(x + 2)^2 + (y - 3)^2 = \left(\frac{11}{2}\right)^2$$

Thus, the graph of this equation is a circle with radius 5½ and center (-2, 3).

5. For $y = 2x^2 + 5x - 2$, find the x–intercepts, the y–intercept and make an appropriate sketch of the graph.

$$a = 2, b = 5, c = -2 \Rightarrow \begin{cases} \text{x-ints} = \dfrac{-5 \pm \sqrt{5^2 - 4(2)(-2)}}{2(2)} = \dfrac{-5 \pm \sqrt{41}}{4} \approx -2.85 \text{ and } 0.35 \\ \text{y-int} = -2 \\ \text{vertex} = \left(\dfrac{-5}{2(2)},\ 2\left(-\dfrac{5}{4}\right)^2 + 5\left(-\dfrac{5}{4}\right) - 2\right) = \left(-\dfrac{5}{4}, -\dfrac{41}{8}\right) \end{cases}$$



6. Find the quadratic function y = f(x) such that, f(-1) = 0, f(0) = -2 and f(1) = 2.

We can write $f(x) = ax^2 + bx + c$, substitute -1, 0 and 1 for x and solve the resulting system of linear equations for a, b and c.

Another approach is to note that since -1 is a zero of this function we know that f must be of the form $a(x - (-1))(x - r)$. Thus, we have $f(x) = a(x + 1)(x - r)$.

Now, $f(1) = 2a(1 - r) = 2$, so $a - ar = 1$. Next, $f(0) = -ar = -2 \Rightarrow a = 3 \Rightarrow r = \frac{2}{3}$.

Therefore, $f(x) = 3(x + 1)(x - \frac{2}{3}) = 3x^2 + x - 2$.

7. Solve for *z* and write your solution in simplified form: $\mathbf{3^{2z-7} = \frac{1}{81}}$.

$$\begin{aligned} 3^{2z-7} &= \frac{1}{81} \\ 3^{2z-7} &= 3^{-4} \\ \log_3(3^{2z-7}) &= \log_3(3^{-4}) \\ 2z - 7 &= -4 \\ 2z &= 3 \\ z &= \frac{3}{2} \end{aligned}$$

8. Solve for x: $\log_7(5x) - \log_7(2 - x) = 1$.

$$\log_7(5x) - \log_7(2 - x) = 1$$

$$\log_7\left(\frac{5x}{2-x}\right) = 1$$

$$7^{\log_7\left(\frac{5x}{2-x}\right)} = 7^1$$

$$\frac{5x}{2-x} = 7$$

$$5x = 14 - 7x$$

$$x = \frac{7}{6}$$

9. How many pounds of an alloy that is 35% copper must be combined with 80 pounds of an alloy that is 75% copper in order to obtain an alloy that is 60% copper. How many pounds of this 60% alloy are produced?

Let x represent the number of pounds of the 35% allow to be added. Expressing the number of pounds of pure copper in the 60% alloy in two ways gives the following equation:

$$0.35x + 0.75(80) = 0.60(x + 80)$$

$$0.35x + 60 = 0.60x + 48$$

$$12 = 0.25x$$

$$48 = x$$

$$x + 80 = 128$$

Thus, combining 48 pounds of an alloy that is 35% copper with 80 pounds of an alloy that is 75% copper we obtain 128 pounds of an alloy that is 60% copper.

10. A tortoise and a hare are involved in a cross-country race. The tortoise travels at the constant rate of 3 miles per hour. The hare overslept so the

tortoise has a 1.5-hour head start. The hare goes at the average rate of 9 miles per hour. How long will it take the hare to overtake the tortoise?

Let t = the number of hours it takes the hare to overtake the tortoise. When the hare catches up, both will have traveled the same distance, so we have

$$9t = 3(t + 1.5).$$

Solving this equation

$$\begin{aligned} 9t &= 3t + 4.5 \\ 6t &= 4.5 \\ t &= 0.75 \text{ or } \frac{3}{4}. \end{aligned}$$

Thus, it will take the three-fourths of an hour or 45 minutes for the hare to overtake the tortoise.

Trigonometry Non-calculator exam.

1. Convert $\frac{17\pi}{6}$ radians to degrees.

$$\frac{17\pi}{6} \text{ radians} = \frac{17\pi}{6} \times \frac{180^\circ}{\pi} = 510^\circ$$

2. If $\cot(x) = -\frac{5}{12}$ and $\sec(x) = \frac{13}{5}$, find $\sin(x)$.

$$\cot(x) = \frac{\cos(x)}{\sin(x)} \Rightarrow \sin(x) = \frac{1}{\sec(x)\cot(x)} = \frac{1}{\left(\frac{13}{5}\right)\left(\frac{-5}{12}\right)} = -\frac{12}{13}$$

3. Find the exact values of sin x and cos x if x = -210°.

$$\cos(-210°) = \cos(150°) = -\cos(30°) = -\frac{\sqrt{3}}{2}$$

$$\sin(-210°) = \sin(150°) = \sin(30°) = \frac{1}{2}$$

4. Verify the identity $\quad$ **$[\sin^4(x) - \cos^4(x)]^2 = 1 - 4\sin^2(x)\cos^2(x)$**

$$\begin{aligned}\left[\sin^4(x) - \cos^4(x)\right]^2 &= \left[\left(\sin^2(x) - \cos^2(x)\right)\left(\sin^2(x) + \cos^2(x)\right)\right]^2 \\ &= \left[\left(-\cos(2x)\right)(1)\right]^2 \\ &= \cos^2(2x) \\ &= 1 - \sin^2(2x) \\ &= 1 - \left(2\sin(x)\cos(x)\right)^2 \\ &= 1 - 4\sin^2(x)\cos^2(x)\end{aligned}$$

5. Find all $x \in [0, 2\pi)$ that satisfy $\tan(x) = \frac{2\sqrt{3}}{3}\sin(x)$.

$$\tan(x) = \frac{2\sqrt{3}}{3}\sin(x) \Rightarrow \frac{\sin(x)}{\cos(x)} = \frac{2}{\sqrt{3}}\sin(x)$$

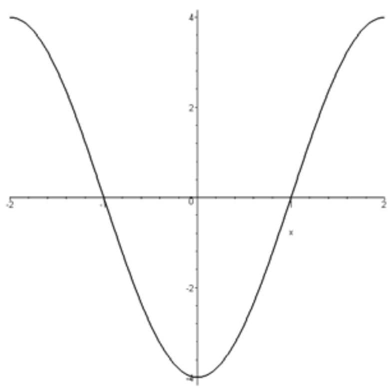
$$\Rightarrow \begin{cases} \sin(x) = 0 \Rightarrow x \in \{0, \pi\} \\ \text{or} \\ \cos(x) = \frac{\sqrt{3}}{2} \Rightarrow x \in \left\{\frac{\pi}{6}, \frac{11\pi}{6}\right\} \end{cases}$$

Thus, $x \in \left\{0, \frac{\pi}{6}, \pi, \frac{11\pi}{6}\right\}$.

6. Write $\sin\left(\arctan\left(\frac{x}{5}\right)\right)$ as an algebraic expression free of trigonometric or inverse trigonometric functions.

Let $\theta = \arctan\left(\frac{x}{5}\right)$. It is convenient to think of θ as an angle in a right triangle with opposite side x, adjacent side 5 and hypotenuse $\sqrt{x^2+5^2}$. Therefore, $\sin(\theta) = \frac{x}{\sqrt{x^2+5^2}}$. The triangle argument is only valid if $x > 0$, but it is easy to verify that this relationship actually holds for any real number x.

7. The graph below is one period of a function of the form $y = a \sin k(x - b)$. Determine the values of a, k and b, where $a > 0$.



Since y “lives” between -4 and 4, it follows that a = 4.

Since x “lives” between -2 and 2, it follows that the period is 4. Thus, $2\pi = 4k \Rightarrow k = \frac{\pi}{2}$.

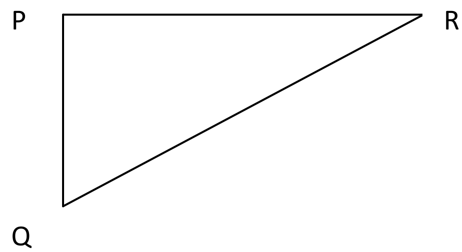
Since sin(0) = 0 and sin is increasing at 0, it follows that this graph corresponds the graph of $y = 4\sin\left(\frac{\pi}{2}x\right)$ shifted 1 unit to the right. Thus, replacing x by x – 1 gives the function we seek, so

b = 1.

a = 4, $k = \frac{\pi}{2}$, b = 1 $\Rightarrow$ $y = 4\sin\left(\frac{\pi}{2}(x-1)\right)$

Trigonometry Calculator portion of the exam.

8. In triangle PQR, m∠P = 90°, m∠Q = 31°, PR = 6.4. Find each unknown length or angle measure.



m∠R = 90° − 31° = 59°

$$\sin(31^\circ) = \frac{PR}{QR} \Rightarrow QR = \frac{PR}{\sin(31^\circ)} \approx \frac{6.4}{0.5150} \approx 12.4$$

$$\tan(31^\circ) = \frac{PR}{PQ} \Rightarrow PQ = \frac{PR}{\tan(31^\circ)} \approx \frac{6.4}{0.6009} \approx 10.7$$

9. Approximate the value of the angle x in the interval (0,π) where sec(x) = -4.15. Show your answer correct to two decimal places.

$$\sec(x) = -4.15 \Rightarrow \cos(x) = \frac{1}{-4.15} \approx 1.81 \text{ radians}$$

10. A triangle has angle A = 12°, angle C = 47° and side c = 25. Find the length of the remaining side a, correct to two decimal places.

Applying the law of sines, we have

$$\frac{\sin(12^\circ)}{a} = \frac{\sin(47^\circ)}{25} \Rightarrow a = \frac{25\sin(12^\circ)}{\sin(47^\circ)} \approx \frac{25(0.2079)}{0.7314} \approx 7.11$$

b. Scoring Guide for the Algebra-Trigonometry Exam

Exams are graded independently by two scorers. Each question is worth five points and the scores for algebra and trigonometry are reported separately. In order to pass, candidates must earn at minimum of 35 points in each area from each scorer. Scores that differ by more than 3 points are rescored jointly; the overall raw score is the average from the two scorers. Raw scores are not released to candidates. Instead, they receive a rubric score of 1, 2 or 3, and a score of 2 is required to pass. Candidates who earn more than 85 for a raw score are awarded a rubric score of 3.

c. Data for Completers (undergraduates and RITE candidates) from the Algebra-Trigonometry Exam

Secondary Education – Mathematics Program Completers (undergraduate and RITE candidates) for years 2008 – 2010 on Algebra-Trigonometry Exam				
Years	Number Taking Exam	Number Passing Exam	Institutional Pass Rate	Mean Score
2007-2008	16	16	100%	79.75
2008-2009	6	6	100%	75.2
2009-2010	8	8	100%	85.9