June 2004

PHILOSOPHY STATEMENT FOR MATH 431: NUMBER THEORY

CATALOG DESCRIPTION

Topics include number systems, divisibility, primes and factorization, Diophantine problems, congruences, and Euler's and Fermat's Theorems.

PURPOSE

Math 431 introduces the students to elementary number theory. A secondary purpose of Math 431 is to introduce/reinforce the craft of proof construction.

MAJOR TOPICS

The major topics are divisibility and factorization, primes, perfect numbers, congruences, Diophantine equations, and Fermat's and Euler's theorems. Other topics (chosen at discretion of the instructor) might include mathematical induction, Pythagorean triples, continued fractions, and ciphers.

PREREQUISITE

Math 2121: Calculus I or Math 247: Calculus; A Short Course.

APPROACH

This course should be geared so that students who possess only the prerequisite have a reasonable chance of success. Concepts should be illustrated with numerical examples and counterexamples wherever possible. Attempts should be made to have students appreciate the purpose of definitions and theorems as they are encountered. Not all theorems need to be rigorously proved; proofs should be selectively chosen in order to help students gain mathematical maturity, not simply to be able to say that the theorems have been proved. Proofs of different types should be analyzed.

CLIENTELE

The students will generally be mathematics majors and mathematics concentrators within the Elementary Education curriculum. MAT candidates may appear occasionally.

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COURSE OUTLINE FOR MATH 431: NUMBER THEORY

TEXT: Vanden Eynden, Elementary Number Theory, Second Edition, 2001.

Chapter/Remarks	Number of Weeks
1. Divisibility	3.5
Do 1.1 (The GDC and LCM),	
1.2 (The Division Algorithm),	
1.3 (The Euclidean Algorithm),	
1.4 (Linear Combinations),	
1.5 (Congruences).	
2. Prime Numbers	2.5
Do 2.1 (Prime Factorization),	
2.2 (The fundamental Theorem of Arithmetic),	
2.3 (The Importance of Power Factorizations),	
2.4 (Prime Power Factorizations),	
2.5 (The Set of Primes is Infinite),	
2.6 (A Formula for γ (n).	
3. Numerical Functions	3.5
Do 3.1 (The Sum of the Divisors),	
3.2 (Multiplicative Functions),	
3.3 (Perfect Numbers)	
3.4 (Mersenne and Fermat Numbers),	
3.5 (The Euler Phi Function),	
3.6 (The Möbius Inversion Formula).	
4. The Algebra of Congruence Classes	2.5
Do 4.1 (solving Linear Congruences),	
4.2 (The Chinese Remainder Theorem),	
4.3 (The Theorems of Fermat and Euler).	
	12
Exams, Quizzes, Review	2
Total	14

Note: If you have any time leftover, you may want to cover the following sections:

- 1.6 (Mathematical Induction)
- 4.5 (Public-Key Cryptography)
- 5.3 (Quadratic Residues)
- 5.4 (Quadratic Reciprocity)
- 7.1 (Pythagorean Triples)
- 7.2 (Sums of Two Squares)
- 7.3 (Sums of Four Squares)